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THE REGULAR CASE OF SHOCK WAVE DIFFRACTION ON A WEDGE PARTLY SUBMERGED IN FLUID*

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The pattern of diffraction at the gas-liquid interface, induced by a system of oncoming shock waves of regular interaction in gas impinging on a wedge submerged in the liquid is considered. The wedge of arbitrary angle has its apex at the unperturbed interface with one of its sides at a nearly straight angle to the liquid unperturbed surface. The liquid is assumed only slightly compressible, which makes it possible to consider the problem in linear formulation by the method proposed in /1−3/.

The normal case of shock wave diffraction on a wedge submerged in liquid with its apex at the unperturbed interface, and the regular case of shock diffraction on a corner of angle close to straight were considered in /3/ and /4/, respectively. The findings of /3/ are extended to the case of an arbitrary angle of incidence of a shock wave in gas in the regular interaction mode.

1. Statement of the problem. A plane shock wave of arbitrary intensity moves at some angle to the interface of two media of greatly different densities. The shock wave is reflected into the lower density medium and refracted into the denser one, while the interface becomes depressed and forms a nearly straight angle with the unperturbed state. This problem was considered in /5/, where a closed system of algebraic equations was obtained for the determination of the unknown flow parameters in regions 0, 1, and 2 (Fig.1).

Here, the diffraction of the described system of waves on a wedge submerged in liquid whose one side adjoins the liquid and the other the gas. The latter may form with the unperturbed free surface level an angle close to the straight one. The perturbed liquid flow contains the diffraction region NLH with adjacent zones with piecewise constant parameters defined in /3/. The perturbed flow behind the reflected shock wave in gas consists of the diffraction region $ABCD$ with adjacent regions ADM and $BHGC$ and of a stream with constant parameters, where the perturbations induced by the wedge in the liquid (Fig.2) do not reach. Flow parameters in region ADM are also constant, and are determined as a regular shock wave reflection from a solid wall /6/. As the basic unperturbed state parameters, we take parameters behind the shock wave reflected from a solid wall whose plane coincides with the unperturbed free surface level. They are: P the pressure, R the density, V_1 the stream velocity parallel to the wall, a the speed of sound, U the normal velocity of the reflected shock wave, and β the angle of the reflected shock wave.

Because of the presence of the small parameter $\varepsilon = R/R_2$, where R_2 is the density of fluid, the problem can be considered in linear formulation. In the first approximation the flow in the liquid is determined by pressure P, as shown in /3/, while the shock wave in gas diffracts at the interface whose shape is known. The gas flow is considered in moving self-similar coordinates

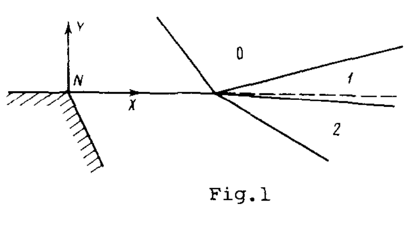

Fig.1

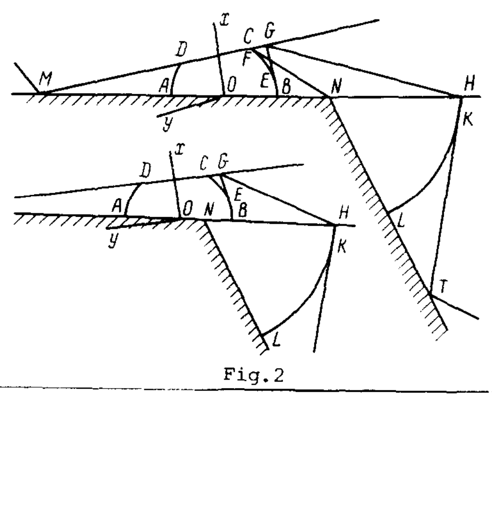

Fig.2

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$$\bar{x} = \frac{X - V_1 t}{at}, \quad \bar{y} = \frac{Y}{at}$$

turned subsequently by the angle $\theta_0 = \pi/2 + \beta$, where (X, Y) are fixed physical coordinates, t is the time, and (x, y) moving and turned self-similar coordinates. The perturbed pressure in terms of polar coordinates (r, θ) then satisfies the equation

$$r^2 (1 - r^2) p_{rr} + p_{\theta\theta} + r (1 - 2r^2) p_r = 0, \; p \to p/(a^2 R) \tag{1.1}$$

within the unit circle of elliptic type and outside it of the hyperbolic type /1/. The arrow denotes equality with accuracy to within the symbol.

2. **The flow of gas outside the diffraction region.** Within region $BHGC$ the flow varies and is induced by the diffraction region of the liquid. To determine perturbed parameters we transform (1.1) to the wave equation /7/

$$p_{\mu\mu} - p_{\theta\theta} = 0, \; \mu = \arccos r^{-1}, \; r > 1 \tag{2.1}$$

whose characteristics are the half-tangents to the Mach circle $r = 1$, facing in opposite directions. The side wave HG coincides with the first set characteristic and, generally, is tangent to the continuation of arc BC into the region ahead of the reflected shock wave. The reflected wave GE coincides with the second set characteristic and is tangent to the Mach circle at point E, it can, also, be tangent to the continuation of arc BC into the region occupied by the liquid and be reflected from the free surface along the first set characteristic.

The solution of Eq. (2.1) in region $BHGE$ is sought in the form $p = \chi(\mu + \theta)$. The form of function χ is determined by the condition at the gas-liquid-solid wall interface boundary f

$$p_\theta (r, -\theta_0) = -r^3 f'' (r - M_1), \; 1 < r < r_H, \; r_H = a_2 / a, \quad M_1 = V_1/a$$

where (a_2 is the speed of sound in the liquid.

For the considered here angles of the wedge a rise of the free surface level takes place in the neighborhood of the wedge apex, hence when $M_1 > 1$ the expression $f''(r - M_1)$ contains the delta function derivative with carrier at the wedge apex, and for $-\pi/2 < \theta_L \leqslant -\pi/4$ also the regularization $(r - M_1)^\alpha$, $\alpha > -2$ which together with the regular part have been defined in /3/. The final solution is of the form

$$p = -\int\limits_{\mu_H - \theta_0}^{\mu+\theta} \sec^3 (\lambda + \theta_0) f'' (\sec (\lambda + \theta_0) - \sec (\theta_F + \theta_0)) \, d\lambda \tag{2.2}$$

$$\mu_H = \arccos r_H^{-1}, \; \theta_F = \arccos M_1^{-1} - \theta_0$$

The solution in region CEG is sought in the form $p = \chi(\mu + \theta) + \varkappa(\mu - \theta)$. Function χ is known from the solution in region $BHGE$, and $\varkappa$ is determined by the condition at the shock wave CG in the polar coordinate system

$$\begin{aligned} & E(\theta) p_r + F(\theta) p_\theta = 0, \; r = m \sec \theta \\ & E(\theta) = (m_1^2 + B) \cos \theta - m (m + A) \sin \theta \operatorname{tg} \theta, \; m_1^2 = 1 - m^2 \\ & F(\theta) = m^{-1} B \cos^2 \theta \operatorname{ctg} \theta - (2m)^{-1} (1 + mA) \sin 2\theta \\ & A = \frac{M_0^2 + 1}{2mM_0}, \quad B = \frac{\gamma + 1}{2} \frac{M_0^2 - 1}{(\gamma - 1) M_0^2 + 2}, \quad M_0 = \frac{U + V}{a_1}, \quad m = \frac{U}{a} \end{aligned} \tag{2.3}$$

where V is the normal component of stream velocity ahead of the reflected shock wave, a_1 is the speed of sound behind the incident shock wave, and γ is the specific heat ratio.

Finally the solution is of the form

$$\begin{aligned} p = & \int\limits_{-\theta_E}^{\mu - \theta} L(\lambda) g^3(\lambda) f''(g(\lambda) - M_1) \, d\lambda - \\ & \int\limits_{\mu_H - \theta_0}^{\mu+\theta} \sec^3 (\lambda + \theta_0) f'' (\sec (\lambda + \theta_0) - M_1) \, d\lambda \end{aligned} \tag{2.4}$$

$$g(\lambda) = \frac{1 + m^2 - 2m \cos \lambda}{\sin (\beta + \lambda) + m^2 \sin (\beta - \lambda) - 2m \sin \beta}, \quad L(\lambda) = \frac{C(\lambda) + D(\lambda)}{C(\lambda) - D(\lambda)}$$

$$C(\lambda) = (1 - m \cos \lambda) [m (m_1^2 + B) \sin^2 \lambda - (m + A) (1 - m \cos \lambda)^2]$$

$$D(\lambda) = (m - \cos \lambda) [(1 + mA) (1 - m \cos \lambda)^2 - m^2 B \sin^2 \lambda]$$

When the wedge is completely submerged in the liquid /3/, a complex system of two functional equations obtains for the determination of the form of the unknown functions χ and $\varkappa$ in region ADM.

Form of the shock wave $x = m + \psi(y)$ in section CG is determined by solving the Cauchy problem for $x = m$

$$y\psi'(y) - \psi(y) = -BM_2^{-1}p(y), \quad \psi(y_G) = BM_2^{-1}p(y_G), \quad M_2 = V/a$$

with the quantity $p(y_G)$ determined in /5/.

3. **The flow of gas in the diffraction region.** The diffraction region is bounded by the reflected shock wave CD, two arcs BC and DA of the Mach circle and for $M_1 > 1$ by the solid wall AB, while for $M_1 < 1$ by the solid wall AN and the liquid free surface NB. The boundary condition on the shock wave is of form (2.3) for $\theta_C < \theta < \theta_D$, $\theta_D = -\theta_C = \arccos m$. The boundary condition on the solid wall is of the form

$$p_n(r, \pi - \theta_0) = 0, \quad 0 < r < 1, \quad p_n(r, -\theta_0) = 0, \quad 0 < r < M_1$$

and on the free surface of the liquid it is

$$p_n(r, -\theta_0) = c_1(r), \quad M_1 \leqslant r < 1$$

where

$$c_1(r) = \vartheta(1 - M_1)[\nu_1\delta'(r - M_1) + \nu_2\delta(r - M_1)] + r^2 f''_1(r - M_1)$$

$$\nu_1 = y_0 M_1^2, \nu_2 = 2y_0 M_1, y_0 = f(-M_1) - \operatorname{tg}\beta_2$$

n is the external normal, ϑ is the Heaviside unit function, δ is the delta function, f_1'' is the regular part of function f'' when $-\pi/4 < \theta < 0$ and the generalized function with singularity $(r - M_1)^\alpha, \alpha > -2$ when $-\pi/2 < \theta \leqslant \pi/4$, $\beta_2 = O(\varepsilon)$, and β_2 is the angle between the wedge wall adjacent to the gas and continuation of the free surface level line taken with the plus sign in the direction of counterclockwise reading.

The boundary condition on arc BC of the Mach circle is obtained from (2.2) and (2.4) with the substitution of variable of the delta function derivative /8/

$$p_\theta(1, \theta) = c_2(\theta), \quad -\theta_0 < \theta < \theta_C, \quad p_\theta(1, \theta) = 0, \quad \theta_D < \theta < \theta_A$$

where

$$c_2(\theta) = -\vartheta(M_1 - 1)[\nu_3\delta'(\theta - \theta_F) + \nu_4\delta(\theta - \theta_F)] - \sec^3(\theta + \theta_0) f''(\sec(\theta - \theta_0) - M_1) - \vartheta(\theta - \theta_E) \times L(\theta) g^3(-\theta) f''(g(-\theta) - M_1)$$

$$\nu_3 = y_0 M_1 (M_1^2 - 1)^{-1}, \quad \nu_4 = \nu_3 (M_1^2 - 2)(M_1^2 - 1)^{-1/2}$$

$$\theta_A = \pi/2 - \beta$$

The two conditions that ensure smoothness of the shock wave front at points C and D and, also, the pressure change along CD by the specified amount, are of the form

$$\int_{-m_1}^{m_1} \frac{p_y}{y} dy = \frac{M_2}{B}(\psi_y(-m_1) - \psi_y(m_1)), \quad \int_{-m_1}^{m_1} p_y \, dy = p_D - p_C \tag{3.1}$$

Application of the Chaplygin transform /1,2/ to Eq.(1.1) reduces it to the Laplace equation, and the diffraction region is transformed in a curvilinear orthogonal tetragon of the plane $\zeta = \rho e^{i\theta}$ bounded by arcs of circles

$$\{\rho = \rho(\theta), \theta_C < \theta < \theta_D\}, \{\rho = 1, \theta_B < \theta < \theta_C, \theta_D < \theta < \theta_A\}$$

and straight-line segment $\{\rho < 1, \theta = \pi - \theta_0, \theta = -\theta_0\}$. The boundary condition for the normal and tangent derivatives of pressure now assume the form

$$ap_n + bp_s = c \tag{3.2}$$

$$\begin{array}{lllll}
a = a(\theta), & b = b(\theta), & c = 0, & \rho = \rho(\theta), & \theta_C < \theta < \theta_D \\
a = 0, & b = 1, & c = 0, & \rho = 1, & \theta_D < \theta < \theta_A \\
a = 1, & b = 0, & c = 0, & \theta = \pi - \theta_0, & 0 < \rho < 1 \\
a = 1, & b = 0, & c = 0, & \theta = -\theta_0, & 0 < \rho < \rho_1 \\
a = 1, & b = 0, & c = c_3(\rho), & \theta = -\theta_0, & \rho_1 < \rho < 1 \\
a = 0, & b = 1, & c = c_2(\theta), & \rho = 1, & \theta_B < \theta < \theta_C
\end{array}$$

where

$$a(\theta)=\sqrt{1-m^2\sec^2\theta}, \qquad b(\theta)=B\operatorname{ctg}\theta-mA\operatorname{tg}\theta$$

$$c_3(\rho)=\vartheta(1-M_1)[\nu_5\delta'(\rho-\rho_1)+\nu_6\delta(\rho-\rho_1)+f_2(\rho)]$$

$$f_2(\rho)=\frac{8\rho^2}{(1+\rho^2)^3}f_1''\left(\frac{2\rho}{1+\rho^2}-M_1\right), \quad \rho(\theta)=\frac{\cos\theta-\sqrt{m^2-\cos^2\theta}}{m}$$

$$\nu_5=2y_0\rho_1^2\frac{1+\rho_1^2}{(1-\rho_1^2)^2}, \qquad \nu_6=4y_0\rho_1\frac{1-4\rho_1^2+\rho_1^4}{(1-\rho_1^2)^3}$$

$$\rho_1=\frac{1-\sqrt{1-M_1^2}}{M_1}$$

and the (n, s) and (x, y) orientations coincide.

Let us map the region bounded by the curvilinear quadrangle of the ζ plane onto the interior of the rectangle $\{0<\sigma<\sigma_0,\ 0<\tau<\pi\}$ of plane $\omega=\sigma+i\tau/4/$. We have

$$\omega=\ln\frac{\zeta-e^{i\theta_2}}{\zeta-e^{i\theta_1}}-i\frac{\theta_2-\theta_1}{2}, \quad \begin{array}{l}\theta_1=\beta_1-\beta\\ \theta_2=\pi-\beta-\beta_1\end{array}$$

$$\sigma_0=\frac{1}{2}\ln\frac{1}{q}, \quad q=\frac{1-\sqrt{M^2-1}\operatorname{tg}\beta}{1+\sqrt{M^2-1}\operatorname{tg}\beta}, \quad \begin{array}{l}\beta_1=\arcsin M^{-1}\\ M=m_1\operatorname{cosec}\beta\end{array}$$

We introduce the analytic function $W(\omega)=p_\sigma-ip_\tau$ transforming by that boundary condition (3.2) to the form of Gilbert's problem in the class of the generalized functions

$$\begin{array}{llll} a_1p_\sigma-b_1p_\tau=d & & & \\ a_1=a_1(\tau), & b_1=b_1(\tau), & d=0, & \sigma=\sigma_0,\ 0<\tau<\pi\\ a_1=1, & b_1=0, & d=0, & \tau=\pi,\ 0<\sigma<\sigma_0\\ a_1=1, & b_1=0, & d=d_1(\tau), & \sigma=0,\ 0<\tau<\pi\\ a_1=1, & b_1=0, & d=d_2(\sigma), & \tau=0,\ 0<\sigma<\sigma_0 \end{array} \tag{3.3}$$

where

$$a_1(\tau)=\operatorname{tg}\beta M\sqrt{M^2-1}(M\cos\tau-m_0)\sin\tau,$$

$$m_0=\sqrt{1-(M^2-1)\operatorname{tg}^2\beta}$$

$$b_1(\tau)=\operatorname{tg}^2\beta m_0^{-2}M^2B(M-m_0\cos\tau)^2-mA(M\cos\tau-m_0)^2$$

$$d_1(\tau)=\vartheta(1-M_1)[\nu_7\delta'(\tau-\tau_1)+\nu_6\delta(\tau-\tau_1)+f_3(\tau)]$$

$$d_2(\sigma)=\vartheta(M_1-1)[\nu_8\delta'(\sigma-\sigma_1)+\nu_4\delta(\sigma-\sigma_1)]-f_4(\sigma)+\Omega(\sigma)$$

$$f_3(\tau)=\frac{\vartheta(\tau_1-\tau)\cos\beta_1}{1+\sin(\tau-\beta_1)}f_2\left(\frac{\cos\tau-\sin\beta_1}{1+\sin(\tau-\beta_1)}\right)$$

$$f_4(\sigma)=\cos\beta_1\frac{(\operatorname{ch}\sigma-\sin\beta_1)^2}{(1-\sin\beta_1\operatorname{ch}\sigma)^3}f''\left(\frac{\operatorname{ch}\sigma-\sin\beta_1}{1-\sin\beta_1\operatorname{ch}\sigma}-M_1\right)$$

$$\Omega(\sigma)=\frac{\vartheta(\sigma-\sigma_2)\cos\beta_1}{\operatorname{ch}\sigma-\sin\beta_1}\,\frac{T(\sigma)+N(\sigma)}{T(\sigma)-N(\sigma)}\varphi^3(\sigma)f''(\varphi(\sigma)-M_1)$$

$$\tau_1=\pi/2+\beta_1-2\operatorname{arcctg}(\rho_1\sec\beta_1+\operatorname{tg}\beta_1)$$

$$\sigma_{1,2}=\frac{1}{2}\ln\frac{1-\cos(\theta_{F,E}-\theta_2)}{1-\cos(\theta_{F,E}-\theta_1)}$$

$$\nu_7=-\nu_5\sec\beta_1[1+\sin(\tau_1-\beta_1)], \nu_8=\nu_3(\sec\beta_1\operatorname{ch}\sigma_1-\operatorname{tg}\beta_1)$$

and $T(\sigma), N(\sigma)$ as well as $\varphi(\sigma)$ are readily found by substituting

$$\sin\theta=\frac{\sin\beta_1\cos\beta\operatorname{ch}\sigma-\cos\beta_1\sin\beta\operatorname{sh}\sigma-\cos\beta}{\operatorname{ch}\sigma-\sin\beta_1}$$

$$\cos\theta=\frac{\cos\beta_1\cos\beta\operatorname{sh}\sigma+\sin\beta_1\sin\beta\operatorname{ch}\sigma-\sin\beta}{\operatorname{ch}\sigma-\sin\beta_1}$$

into the expressions for $C(\theta), D(\theta)$ and $g(-\theta)$ in (2.4).

We write conditions (3.1) in the ω plane as

$$\int_0^\pi\frac{M-m_0\cos\tau}{M\cos\tau-m_0}p_\tau\,d\tau=G(\psi_y(m_1)-\psi_y(-m_1)) \tag{3.4}$$

$$\int_0^\pi p_\tau\,d\tau=p_D-p_C, \qquad G=mm_0M_2M^{-1}B^{-1}\operatorname{ctg}\beta$$

We map the rectangle of plane ω onto the upper half-plane of plane $w=u+iv$

$$w = \frac{\vartheta_2(0,q)\,\vartheta_2(-i\omega,q)}{\vartheta_3(0,q)\,\vartheta_3(-i\omega,q)}$$

where $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4$ are elliptic theta functions /9/. The intervals $(-\infty, -1)$ and $(1, +\infty)$ correspond to the shock wave, and $(-k, k)$, to the wall, with k denoting the elliptic integral modulus. The index of Hilbert's problem (3.3) in the class of functions integrable at points $w=\pm 1$ is unity.

We represent the canonical function in the form

$$Z(w) = Z_1(w)\,Z_2(w)$$

$$Z_1(w) = (w^2-1)^{-1/2} = i\frac{\vartheta_3(0,q)\,\vartheta_3(-i\omega,q)}{\vartheta_4(0,q)\,\vartheta_4(-i\omega,q)}$$

$$Z_2(w(\omega)) = \exp\Big(-\sum_{n=1}^{\infty}\Big[4-\sum_{j=1}^{4}\Big(\frac{h_j-1}{h_j+1}\Big)^n\Big]\frac{\operatorname{ch} n\omega}{n \operatorname{sh} n\sigma_0}\Big)$$

$$h_j = \Big(\frac{M+m_0}{M-m_0}\Big)^{1/2}\big(l_{1,2}\pm\sqrt{l_{1,2}^2-1}\big),$$

$$l_{1,2} = \frac{m_1^2 \pm (m_1^4 - 4mm_1^2AB - mB)^{1/2}}{2(m_1^2A - mB)}$$

where $Z_1(w)$ has a piecewise constant argument at the boundary and eliminates discontinuities at points $w=\pm 1$ /2/, and $Z_2(w)$ satisfies the condition on the shock wave image /4/.

The solution of Hilbert's problem (3.3) is of the form /10,11/

$$W(\omega(w)) = Z(w)\Big(D_0 + D_1 w + \frac{1}{\pi i}\int_{-k}^{k}\frac{d_1(\tau(s))}{Z(s)}\frac{ds}{s-w} + \frac{1}{\pi i}\int_{k}^{1}\frac{d_2(\sigma(s))}{Z(s)}\frac{ds}{s-w}\Big)$$

Let us also write down the solution in the ω plane, and use the property of the delta function and of its derivative as the density of the Cauchy type integral /2,12/

$$\begin{aligned}
W(\omega) = z(\omega)\Big[&D_0 + D_1 w(\omega) + \\
&\vartheta(1-M_1)\Big(\frac{\nu_9}{\pi i}\frac{d}{ds}\frac{1}{Z(s)(s-w(\omega))}\Big|_{s=u_1} + \\
&\frac{\nu_{10}}{\pi i}\frac{1}{u_1 - w(\omega)} + \frac{1}{\pi i}\int_{\tau_1}^{0}\frac{f_3(s)}{z(s)}U(0,s,\omega)\,ds\Big) + \\
&\vartheta(M_1-1)\Big(\frac{\nu_{11}}{\pi i}\frac{d}{ds}\frac{1}{Z(s)(s-w(\omega))}\Big|_{s=u_2} + \\
&\frac{\nu_{12}}{\pi i}\frac{1}{u_2 - w(\omega)}\Big) + \frac{1}{\pi i}\int_{\sigma_1}^{\sigma_0}\frac{\Omega(s)}{z(s)}U(s,0,\omega)\,ds - \\
&\frac{1}{\pi i}\int_{0}^{\sigma_0}\frac{f_4(s)}{z(s)}U(s,0,\omega)\,ds\Big]
\end{aligned} \tag{3.5}$$

where

$$U(0,s,\omega) = \frac{u_s(0,s)}{u(0,s)-w(\omega)}, \qquad U(s,0,\omega) = \frac{u_s(s,0)}{u(s,0)-w(\omega)}$$

$$u(0,\tau) = \sqrt{k}\,\frac{\vartheta_2(\tau,q)}{\vartheta_3(\tau,q)}, \qquad u(\sigma,0) = \sqrt{k}\,\frac{\vartheta_4(\sigma,q')}{\vartheta_3(\sigma,q')}$$

$$u_\tau(0,\tau) = -\frac{2K\sqrt{kk'}}{\pi}\frac{\vartheta_1(\tau,q)\,\vartheta_4(\tau,q)}{\vartheta_3^2(\tau,q)}, \qquad \sqrt{k} = \frac{\vartheta_2(0,q)}{\vartheta_3(0,q)}$$

$$u_\sigma(\sigma,0) = \frac{2K\sqrt{kk'}}{\pi}\frac{\vartheta_1(\sigma,q')\,\vartheta_2(\sigma,q')}{\vartheta_3^2(\sigma,q')}, \qquad q' = \exp\frac{\pi^2}{\ln q}$$

$$u_{\tau\tau}(0,\tau) = -\frac{4K^2\sqrt{kk'}}{\pi^2}\frac{\vartheta_2(\tau,q)}{\vartheta_3^3(\tau,q)}[\vartheta_4^2(\tau,q) + k\vartheta_1^2(\tau,q)]$$

$$u_{\sigma\sigma}(\sigma,0) = \frac{4K^2\sqrt{kk'}}{\pi^2}\frac{\vartheta_4(\sigma,q')}{\vartheta_3^3(\sigma,q')}[\vartheta_2^2(\sigma,q') - k\vartheta_1^2(\sigma,q')]$$

$$z(\omega) = Z(w(\omega)),\quad u_1 = u(0,\tau_1),\quad u_2 = u(\sigma_1,0),\quad k^2 + k'^2 = 1$$

$$\nu_9 = -\nu_7 u_\tau^2(0,\tau_1),\quad \nu_{10} = [\nu_6 u_\tau(0,\tau_1) - \nu_7 u_{\tau\tau}(0,\tau_1)]\,Z^{-1}(u_1)$$

$$\nu_{11} = \nu_8 u_\sigma^2(\sigma_1,0),\quad \nu_{12} = [\nu_1 u_\sigma(\sigma_1,0) - \nu_3 u_{\sigma\sigma}(\sigma_1,0)]\,Z^{-1}(u_2)$$

K is the complete elliptic integral for the modulus of k /9/, and the real constants D_0 and

D_1 are obtained from conditions (3.4).

The determination of pressure using (3.5) enables us to establish the shock wave form using the pressure distribution along its front /3/. It is evident from (3.5) that pressure distribution has a singularity of the first order pole at the wedge apex when $M_1 < 1$ and at the tangency point of the characteristic issuing from the wedge apex with the Mach circle when $M_1 > 1$. Then the pressure has an integrable singularity, since all integrals are taken in the sense of their principal values. A singularity of order higher than the logarithmic encountered in problems of diffraction on solid walls /1,2,4/, are due to the pliability of the liquid free surface.

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